THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050A Extra Tutorial 2

1. Let f and g be uniformly continuous on $A \subseteq \mathbb{R}$. If f,g are both bounded on A, show that fg is uniformly continuous on A. Can the boundedness condition be dropped?

2. Prove that if f is uniformly continuous on a bounded subset A of $\mathbb{R}$, then f is bounded on A.

3. Suppose $I \subseteq \mathbb{R}$ is a closed and bounded interval, $f : I \to \mathbb{R}$ is continuous and positive on I. Show that $1/f$ is uniformly continuous on I. Can the conditions on I be dropped?

4. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function. Suppose $\lim_{x\to\infty} f(x) = l \in \mathbb{R}$. Show that f is uniformly continuous.

5. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic on $\mathbb{R}$ if there exists a number $p > 0$ such that $f(x + p) = f(x)$ for all $x \in \mathbb{R}$. Prove that a continuous periodic function on $\mathbb{R}$ is bounded and uniformly continuous on $\mathbb{R}$.